Option Book Management

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Option Book Management

OVERVIEW

The task of managing books of swaps, caps, floors and swaptions has been raised to a science over the past few years, and primary research continues to bring new tools with which book managers do a better and better job.

Manager of swap books and the traders who work with them have to satisfy many different masters:

1. Clients and swap marketers whose deals must be priced and hedged.
2. Trading managers who want the book to be profitable.
3. Risk managers who want to know the *Value at Risk* of all the positions taken on the book and the return against the normal risk taken.
4. Senior managers who want no surprises in the form of losses due to "extraordinary" rate movements.

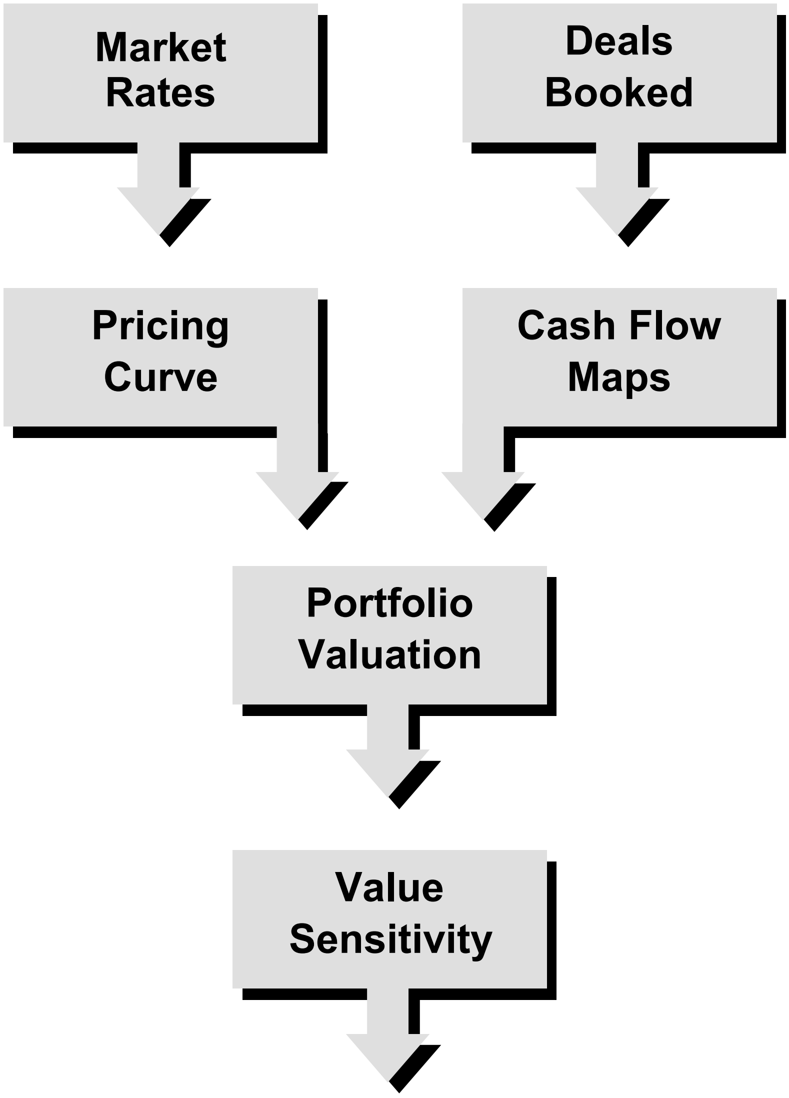
Systems which quantify trades and their changing values have become very sophisticated, and increasing efforts are underway to integrate trading ("front office"), record-keeping ("back office") and risk management ("middle office"). Systems sellers also come closer and closer to being able to accommodate new instruments without re-writing the entire system.

In this module we will examine the task of managing a book of interest rate options. We will describe this task as essentially that of a portfolio manager.

The value of the portfolio is subject to constant changes in interest rates and volatility, and the portfolio is changing constantly as new deals are done and old deals come off.

What drives the pricing decisions of the manager of such a portfolio? What leads him to prefer certain trades and run away from others? How does he hold this shifting bundle of cash flows together?

The management of interest rate options is similar to that of interest rate swaps with the addition of greater interest rate risk and volatility risk. The process is similar, too:



1. The first step is to identify the most liquid market instruments which can be used to manage the risk of the book. This is normally some combination of deposits, 3-month deposit futures and interest rate swaps.

2. Liquid market rates are used to calculate a pricing curve (sometimes called a *discount function*) which can be used to value everything on the book or proposed to the book.

3. Every deal booked is entered into a database which keeps track of the date and amount of every cash flow in every transaction on the book.

4. Instrument cash flows are mapped to the dates of the pricing curve. This is normally done using sophisticated interpolation methods which have the effect of splitting the actual instrument cash flows among the dates on the pricing curve.

5. Mapping the cash flows allows the book to be valued.

6. The book's value sensitivity to interest rate and volatility changes is measured and adjusted using liquid instruments if desired.

The focus of this module lies on the very last step: testing and hedging the book's sensitivity to changing interest rates and volatility levels. This task is the most difficult — and interesting — aspect of managing option books.

INTEREST RATE OPTION PRICING FUNDAMENTALS

Caps and Floors

Caps and floors are options on interest rate swap LIBOR resets.

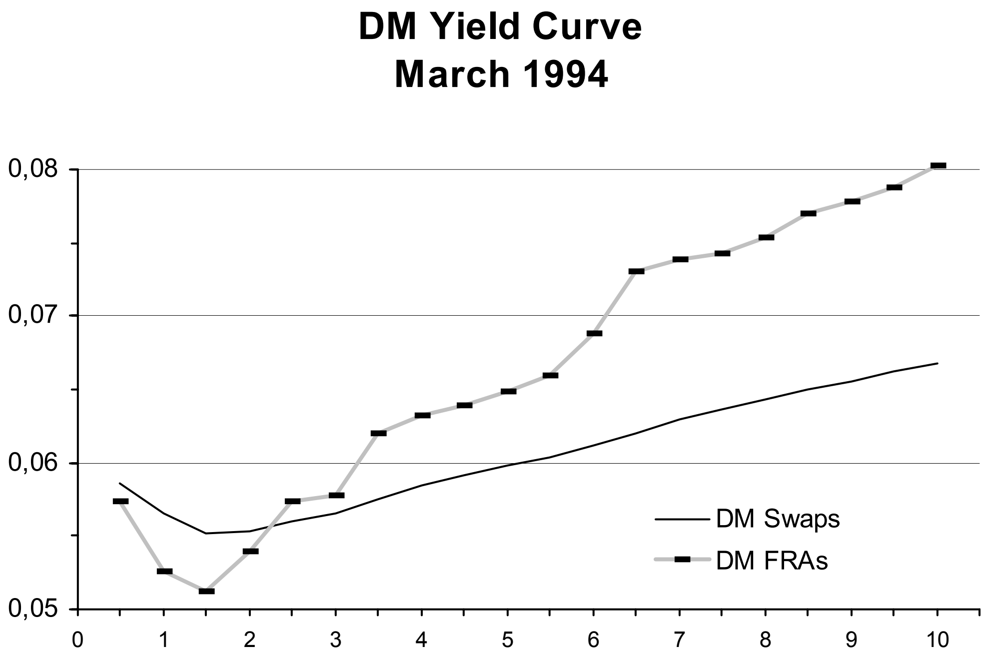
The money for a cap or floor is thus basically the swap rate — with several adjustments:

1. The main structural difference between swaps and caps and floors is that swaps almost always include the floating-rate for the period starting on the day the swap begins, while caps and floors normally do not.

2. In DM and many other currencies, swaps are quoted assuming the fixed rate payments are made once each year. Cap and floor strikes are quoted with the same compounding convention as the LIBOR rate underlying them: 6-month LIBOR is often the underlying rate.

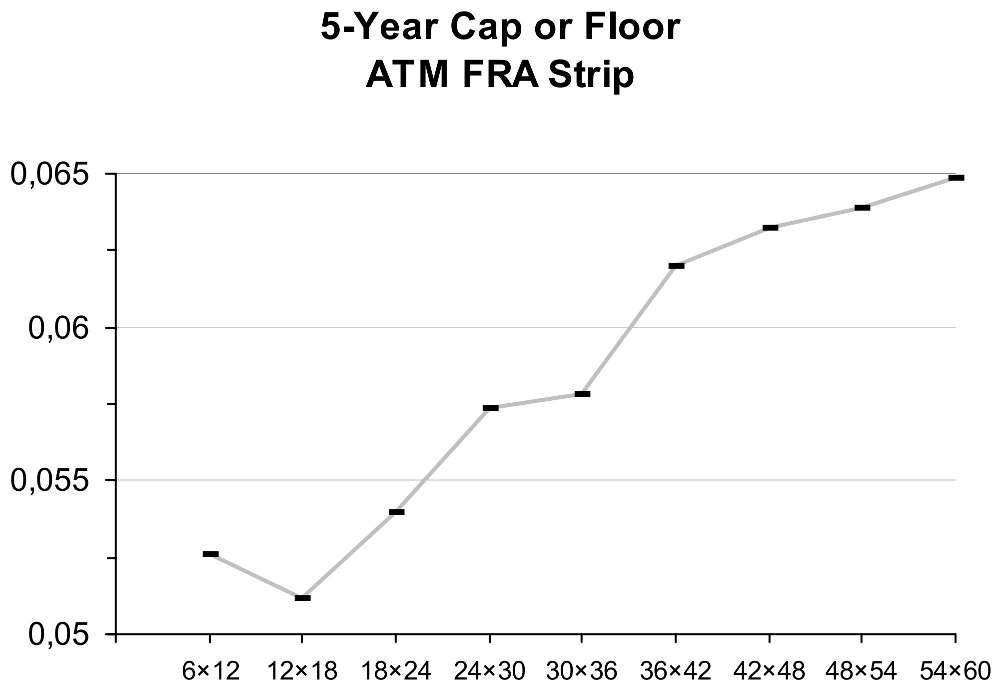
3. Finally, in order to be compared directly to LIBOR rates, which are quoted on a money market basis of actual days in a year of 360 days, cap and floor strikes are normally quoted according to the same convention.

In March 1994 the DM yield curve stood as shown below:



Just as the swap is the economic equivalent of the FRA strip, so, too, are the cap and floor the economic equivalent of options on the strip of successive FRAs.

For a 5-year cap or floor on 6-month LIBOR, therefore, we can say that the money for each successive FRA cap is the FRA itself:



Pricing a 5-Year Cap
To price a 5-year cap on 6-month LIBOR, we have to compare the strike rate of the cap to each of the successive FRAs in order to calculate the price of each FRA option.

The ATM cap rates are calculated in the following way:

$$\text{ATM Cap} = \frac{\sum_{t=2}^{n} \text{FRA}_t \times \text{Days}_t \times \text{PVf}_t}{\sum_{t=2}^{n} \text{PVf}_t \times \text{Days}_t}$$

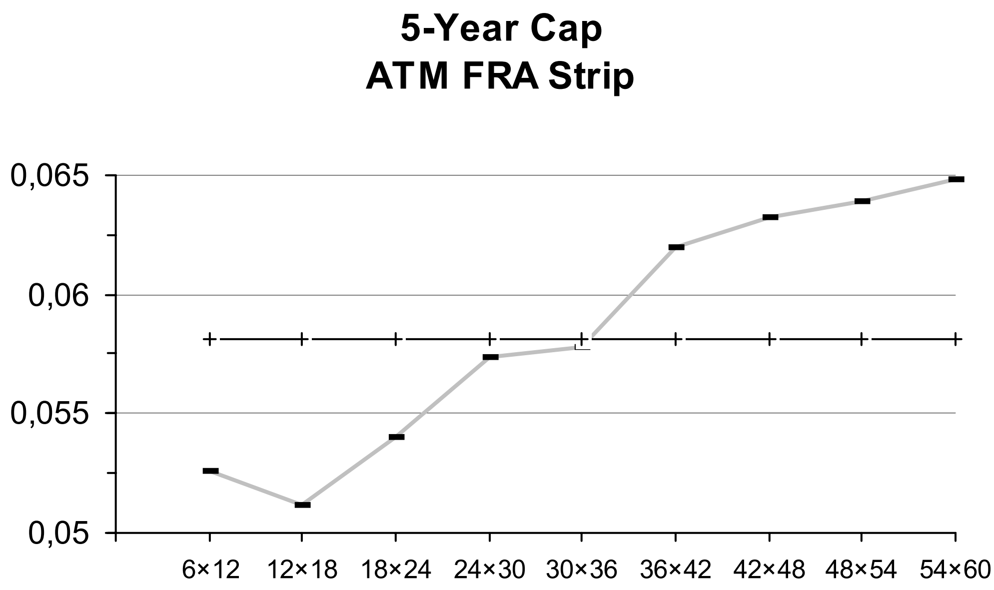
In other words, the ATM cap rate is equal to the PV of the floating-rate cash flows divided by the sum of the relevant PV factors weighted by the number of days in each 6-month period.

The formula can ignore the 360 above and below because it can be factored out of every term.

Using the formula above we can calculate the ATM cap strikes:

Date	Days	PVf's	Swaps	FRAs	ATM Caps & Floors	Swaps (A/360)
9-Mar-94		1,000000				
9-Sep-94	184	0,971537	5,8594%	5,7321%		
9-Mar-95	181	0,946505	5,6518%	5,2600%	5,2600%	5,5011%
9-Sep-95	184	0,922372	5,5145%	5,1191%	5,1899%	5,3763%
9-Mar-96	182	0,897858	5,5381%	5,4006%	5,2581%	5,3821%
9-Sep-96	184	0,872275	5,6007%	5,7382%	5,3740%	5,4499%
9-Mar-97	181	0,847633	5,6600%	5,7823%	5,4505%	5,5010%
9-Sep-97	184	0,821597	5,7497%	6,2000%	5,5674%	5,5930%
9-Mar-98	181	0,796284	5,8400%	6,3227%	5,6651%	5,6744%
9-Sep-98	184	0,771100	5,9089%	6,3900%	5,7471%	5,7452%
9-Mar-99	181	0,746745	5,9800%	6,4869%	5,8190%	5,8091%

For a 5-year cap struck at 5,81% (the ATM 5-year cap strike), we have the following picture:



Black-Scholes Pricing Model
We can price caps and floors on each of the successive FRAs using a modified version of the Black-Scholes option pricing model.

The Black-Scholes derived formula for a single-period cap (sometimes called a "caplet") is as follows:

$$\text{Cap Premium} = \text{Principal} \times \text{PVf}_{\text{Mat.}} \times \left[\text{FRA} \times N(d_1) - \text{Cap Strike} \times N(d_2)\right] \times \frac{\text{Days}}{360}$$

Where:

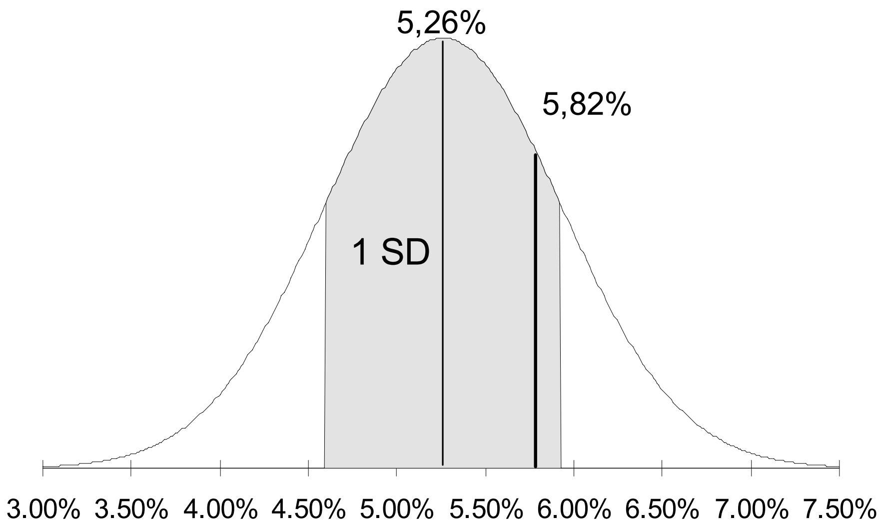
$$d_1 = \frac{\ln\left(\dfrac{\text{FRA}}{\text{Cap Strike}}\right) + \dfrac{\sigma^2 \times t}{2}}{\sigma \times \sqrt{t}}$$

$$d_2 = d_1 - \sigma \times \sqrt{t}$$

Principal =	Notional principal of the cap
$\text{PVf}_{\text{Mat.}}$ =	Present value factor through the end of the forward 3-month or 6-month period
FRA =	The market value of the floating rate for the relevant 3-month or 6-month period
N(...) =	The normal standard distribution function [NORMSDIST(...) in Excel]
ln(...) =	The natural logarithm function. Measures the percent difference between the FRA and the cap strike on a continuously compounded basis.
σ =	Annualized 3- or 6-month LIBOR volatility
t =	Time to expiry for each cap or floor expressed as a fraction of a year

Each FRA cap is priced separately. For each, we are pricing the likelihood that the market level of 6-month LIBOR exceeds the cap strike.

This might be pictured as follows:



In the picture above, we see the caplet for the 6×12 period:

- The strike is set at 5,82%
- The FRA (the "money") is 5,26%
- One standard deviation up and down at 18% volatility is 4,59% and 5,93%

The first part of the cap price formula above, FRA × N(d1), gives us the probability-weighted payoff of the cap, based on the shaded area to the right of the strike line.

The second part of the cap price formula, Strike × N(d2), gives us the probability that the cap will be exercised, based on the shaded area between the FRA and the strike.

In the same way, we can calculate the price of either a cap or a floor for each 6-month period.

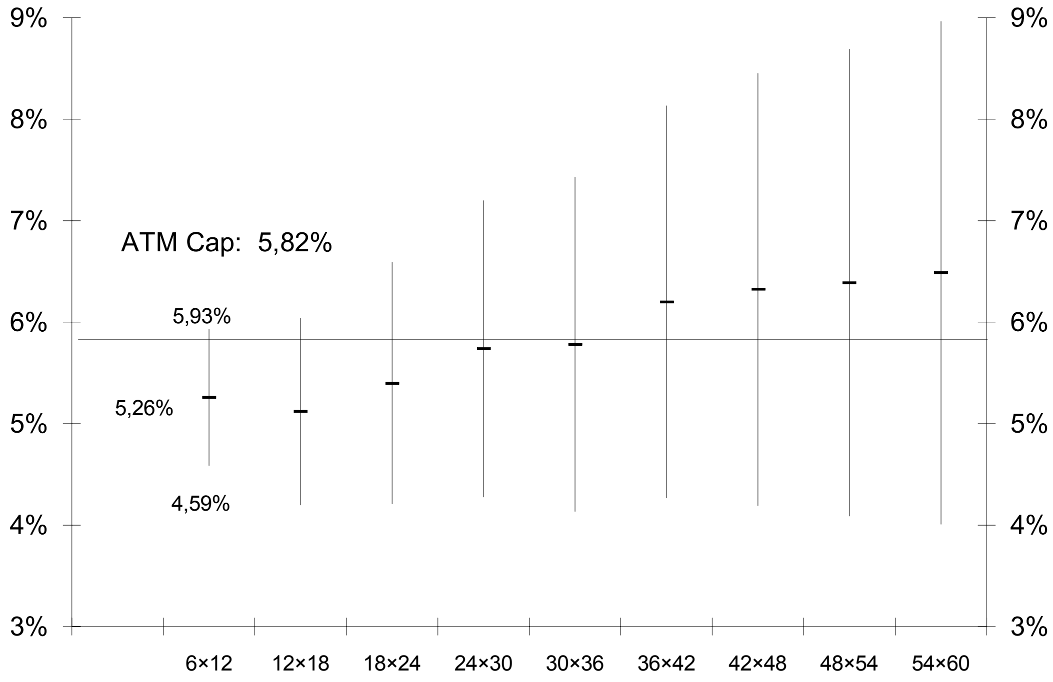
Doing so produces the following results:

Period	Days	Strike	FRA	PV Factor	Volatility	Years	Cap Premium	Floor Premium
0×6	184			0,971537				
6×12	181	5,8190%	5,2600%	0,946505	18,00%	0,504110	0,04%	0,31%
12×18	184	5,8190%	5,1191%	0,922372	18,00%	1,000000	0,06%	0,39%
18×24	182	5,8190%	5,4006%	0,897858	18,00%	1,502732	0,14%	0,33%
24×30	184	5,8190%	5,7382%	0,872275	18,00%	2,000000	0,24%	0,28%
30×36	181	5,8190%	5,7823%	0,847633	18,00%	2,504110	0,27%	0,29%
36×42	184	5,8190%	6,2000%	0,821597	18,00%	3,000000	0,40%	0,24%
42×48	181	5,8190%	6,3227%	0,796284	18,00%	3,504110	0,44%	0,23%
48×54	184	5,8190%	6,3900%	0,771100	18,00%	4,000000	0,47%	0,24%
54×60	181	5,8190%	6,4869%	0,746745	18,00%	4,504110	0,49%	0,24%
							2,5542%	2,5542%

The premiums for ATM caps and floors are equal.

These results make sense when compared to the relationship of the ATM cap and floor strike to the FRA strip:

- Where the cap strike is out of the money (i.e. above the FRA), the cap is relatively cheap.
- Where the cap strike is in the money, the cap is relatively expensive.
- The first two periods place the cap fairly far out of the money, so the first two “caplets” are relatively cheap.



Swaptions

An option on a swap is priced based on possible changes in future levels of LIBOR, just like a cap.

Unlike a cap, however, which offers multiple options on successive LIBOR resets, the swaption offers a single option on a whole series of LIBOR resets.

Because a swaption is an option to pay or receive LIBOR on a swap beginning in the future, the “money” for a swaption is the forward swap rate.

The volatility to be used is the amount of change in each of the FRAs across the strip over the life of the option, i.e. until the beginning of the forward period.

Since there is only one option, swaptions are normally cheaper than caps.

We can compare this by pricing a 3-year cap beginning in two years and comparing it to an option on a 3-year swap beginning in two years, a “2 into 3” swaption.

The price of the forward cap can be computed as above:

Period	Days	Strike	FRA	PV Factor	Volatility	Years	Cap Premium	Floor Premium
24×30	184	6,1382%	5,7382%	0,872275	18,74%	2,000000	0,20%	0,38%
30×36	181	6,1382%	5,7823%	0,847633	18,74%	2,504110	0,23%	0,38%
36×42	184	6,1382%	6,2000%	0,821597	18,98%	3,000000	0,35%	0,33%
42×48	181	6,1382%	6,3227%	0,796284	18,98%	3,504110	0,39%	0,32%
48×54	184	6,1382%	6,3900%	0,771100	15,54%	4,000000	0,36%	0,26%
54×60	181	6,1382%	6,4869%	0,746745	15,54%	4,504110	0,38%	0,25%
							1,9066%	1,9066%

The price of the swaption can be calculated using a similar modified Black-Scholes model:

$$\text{Swaption Premium} = \text{Prin.} \times \text{PVf}_{\text{Exp.}} \times \text{Mod.Dur.} \times \left[\text{Fwd} \times N(d_1) - \text{Strike} \times N(d_2)\right]$$

Where:

$$d_1 = \frac{\ln\left(\frac{\text{Fwd}}{\text{Strike}}\right) + \frac{\sigma^2 \times t}{2}}{\sigma \times \sqrt{t}}$$

$$d_2 = d_1 - \sigma \times \sqrt{t}$$

Principal =	Notional principal of the swaption
PVf_{Exp} =	Present value factor through the expiry of the swaption
Mod. Dur. =	Modified duration of the forward swap (converts yield volatility to a price)
Fwd =	The market value for the forward swap rate
N(...) =	The normal standard distribution function [NORMSDIST(...) in Excel]
ln(...) =	The natural logarithm function. Measures the percent difference between the Fwd and the swaption strike on a continuously compounded basis.
σ =	Annualized swap rate volatility
t =	Time to expiry of the swaption expressed as a fraction of a year

In this case the forward swap rate is 6,3210%.

The 2-year swaption volatility corresponding to the FRA volatilities above is 18,5%.

The ATM 2×3 swaption would cost 1,4549% of the notional principal.

This compares with the cap cost shown above of 1,9066%.

OPTION PRICE SENSITIVITIES

To get a feel for the sensitivity of interest rate option prices as a function of the relevant variables (interest rates, volatility and time), we will use a very simple single-period cap, an option on a single FRA.

Simple Single-Period Cap

The value of a cap is a function of several variables:

1. The level of the FRA
2. Volatility in the FRA
3. Time to maturity

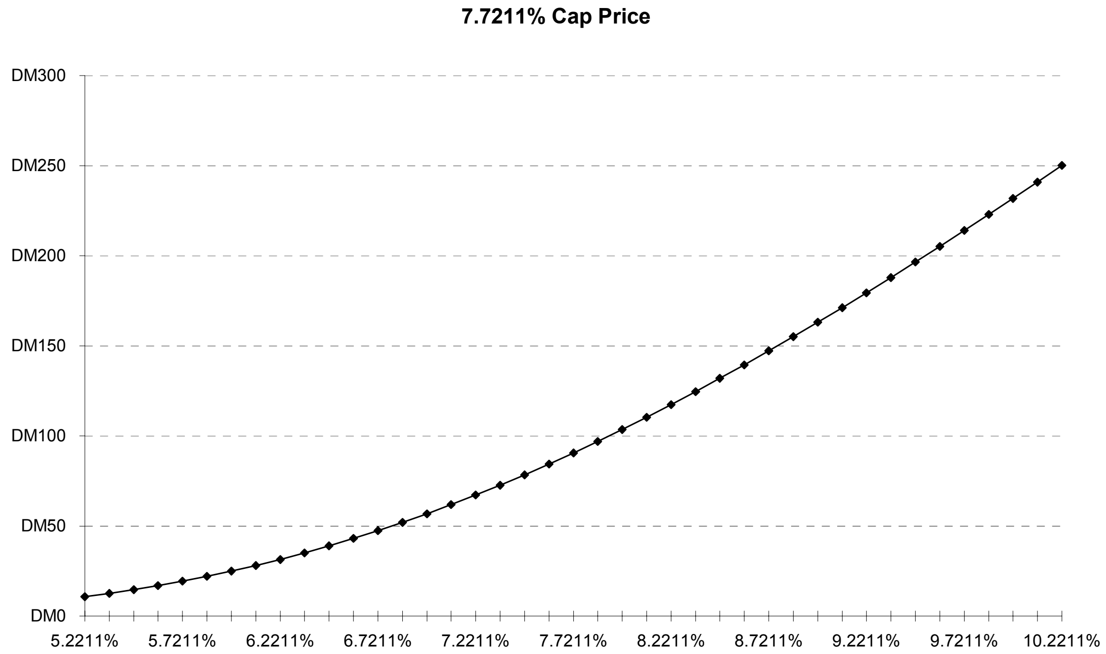
As each of these variables change, the cap price also changes. Cap price sensitivities measure the ratio of cap price change to changes in one of the variables. In the example following the sensitivities are all expressed in DM per one unit change in the variable.

Relevant Market Information

Position	Long
Face Value	DM25,000
Strike	7.7211%
FRA	7.7211%
Maturity	5 years
Days	184 days
PV Factor	0.703507
Volatility	15.50%
Time	4.4962
Value	DM90.5967

Delta

The primary determinant to the price of the option is the level of the underlying, in this case the FRA. This is evident on the following graph:



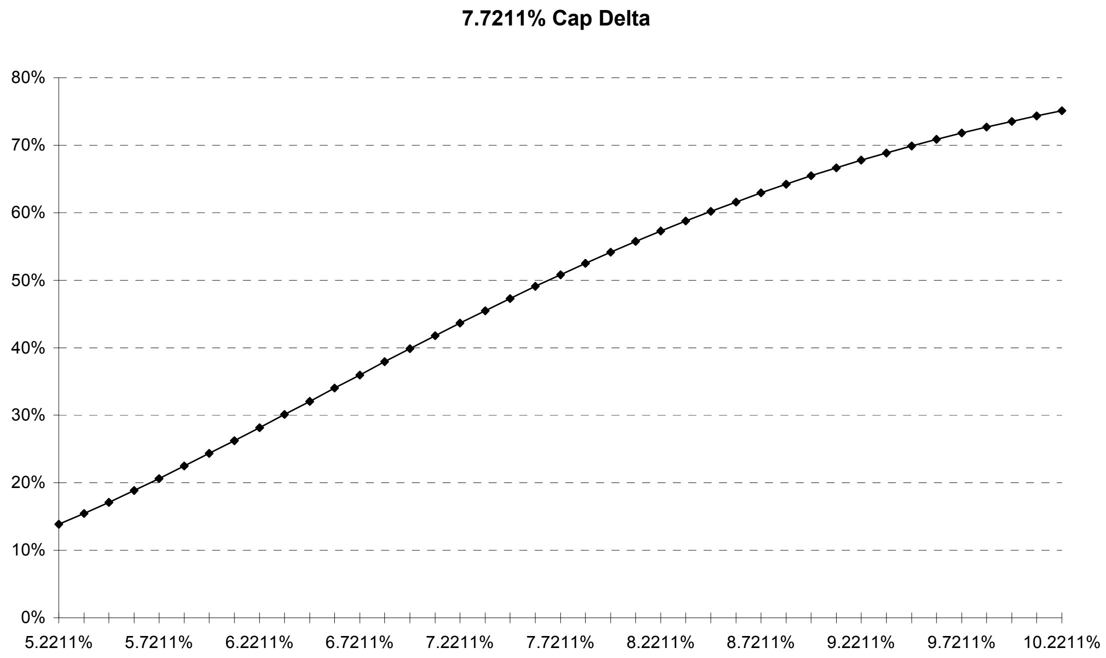
The sensitivity of the option premium with respect to movement in the underlying is known as the delta (Δ) of an option. Delta, from the Greek meaning change.

Deltas for caps are positive, for floors deltas are negative.

- **The equivalent FRA position needed to replicate the payoff of a cap is equal to the delta of the option.**

Deltas for caps vary between 0 and 1. For deep in the money options delta approaches 1 and for deep out of the money options delta approaches 0.

Deltas for floors vary between 0 and -1. For deep in the money options, delta approaches -1 and for deep out if the money options delta approaches 0.



Formula for Delta

For a caplet, delta for a 1 basis point change in the FRA may be calculated as follows:

$$\Delta = N(d_1) \times \text{Principal} \times \frac{\text{Days}}{360} \times \text{PVf}_{\text{Mat.}} \times 0.01\%$$

Where: $$d_1 = \frac{\ln\left(\frac{\text{FRA}}{\text{Cap Strike}}\right) + \frac{\sigma^2 \times t}{2}}{\sigma \times \sqrt{t}}$$

N(...) = The normal standard distribution function [NORMSDIST(...) in Excel]

Principal = Notional principal of the cap

$\text{PVf}_{\text{Mat.}}$ = Present value factor through the end of the forward 3-month or 6-month period

FRA = The market value of the floating rate for the relevant 3-month or 6-month period

ln(...) = The natural logarithm function. Measures the percent difference between the FRA and the cap strike on a continuously compounded basis.

σ = Annualized 3- or 6-month LIBOR volatility

t = Time to expiry for each cap or floor expressed as a fraction of a year

Delta-Neutral Hedging
Delta neutral hedging is a strategy whereby changes in the option position due to a change in the underlying are offset by a change in the underlying position.

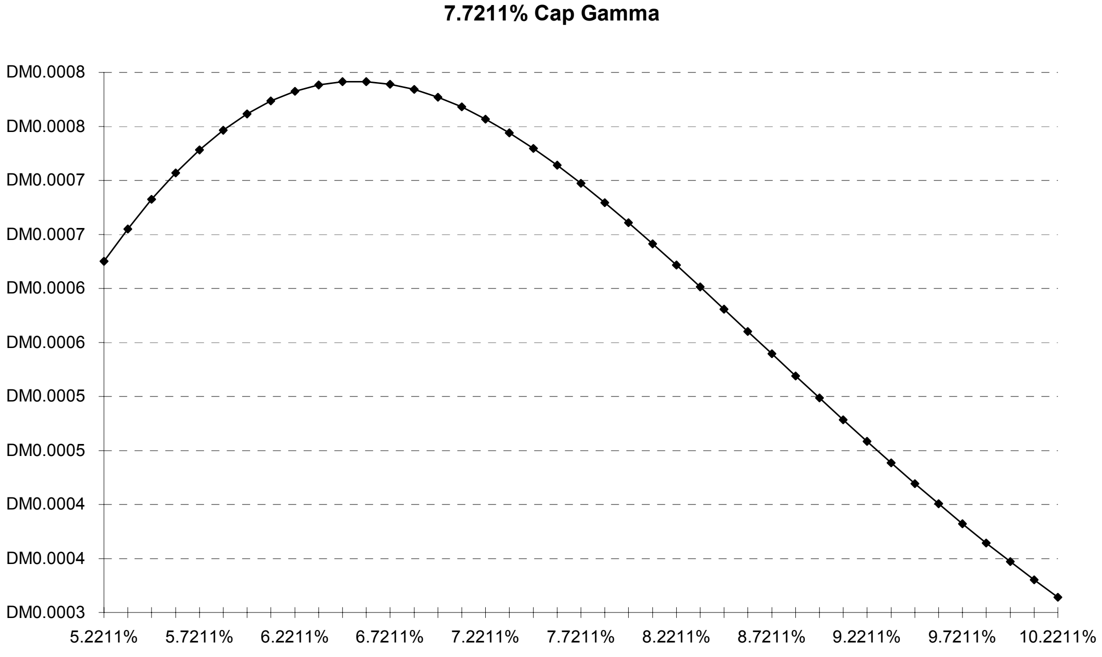
Gamma

As the FRA moves the delta changes. Delta is not a static measurement, but is dependent on the level of interest rates.

The gamma γ statistic measures the change in the delta, or the second order effect of the change in an option price due to a change in the underlying.

Gamma is greatest for at the money options.

The greatest change in delta occurs at the money. This is where the most uncertainty is, since the option can move in or out of the money with a slight shift in the underlying.



For the buyer of options — both caps and floors — gamma is always positive.

For caps, as the FRA increases, delta increases.

For floors, as the FRA increases, delta becomes less negative or more positive.

The price of a cap changes more rapidly for rising prices than the delta would suggest and falls less slowly for falling prices than the delta would suggest.

The price of a floor falls more slowly due to an increase in spot than delta would suggest and increases more rapidly due to a drop in the spot, than delta would suggest.

Formula for Gamma
For a caplet, gamma for a 1 basis point change in the FRA may be calculated as follows:

$$\Gamma = \frac{N'(d_1)}{FRA \times \sigma \times \sqrt{t}} \times Principal \times \frac{Days}{360} \times PVf_{Mat.} \times 0.01\%^2$$

Where: $$N'(d_1) = \frac{\exp\left(-\frac{(d_1{}^2)}{2}\right)}{\sqrt{2 \times \pi}}$$

Principal =	Notional principal of the cap
$PVf_{Mat.}$ =	Present value factor through the end of the forward 3-month or 6-month period
FRA =	The market value of the floating rate for the relevant 3-month or 6-month period
exp(...) =	The exponent function, also expressed as *e.*
σ =	Annualized 3- or 6-month LIBOR volatility
t =	Time to expiry for each cap or floor expressed as a fraction of a year

Gamma is valuable to option buyers. Gamma is dangerous for option sellers.

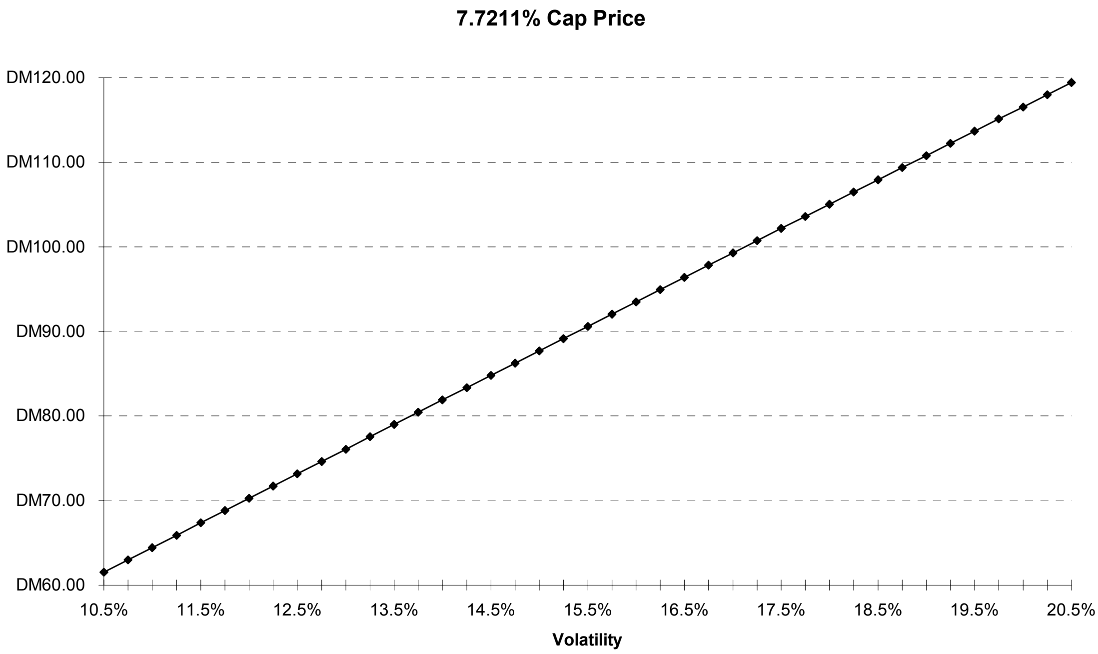
Delta-Neutral Hedging and Gamma
If gamma is small, delta only changes slowly and adjustments to keep a portfolio delta neutral are made relatively infrequently.

If gamma is large, delta is highly sensitive to a change in the underlying asset, and adjustments to keep the portfolio delta neutral have to be made fairly frequently.

Vega
Vega, also referred to as Kappa, is the rate of change of the option value with respect to a change in the volatility of the underlying.

If vega is high the option is sensitive to changes in volatility.

Vega is always positive for buyers of options and negative for sellers of options.



Vega is usually defined for a 1% increase in volatility.

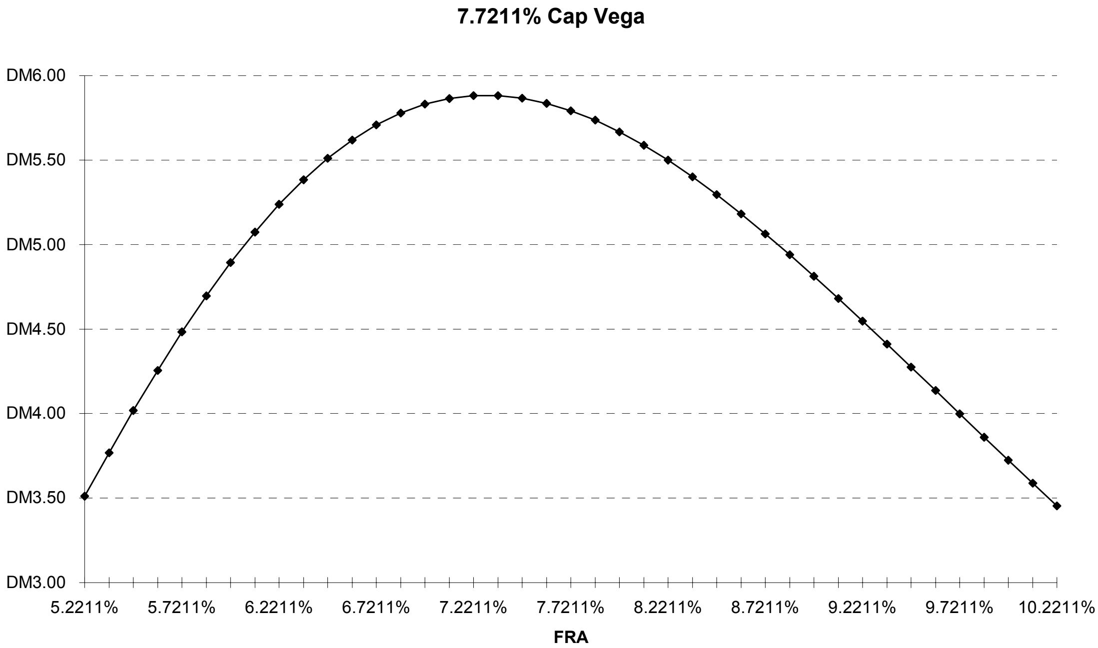
Formula for Vega
For a caplet, vega for a 1% change in volatility may be calculated as follows:

$$\text{Vega} = N'(d_1) \times \text{FRA} \times \sqrt{t} \times \text{Principal} \times \frac{\text{Days}}{360} \times \text{PVf}_{\text{Mat.}} \times 1\%$$

Where: $$N'(d_1) = \frac{\exp\left(-\frac{(d_1{}^2)}{2}\right)}{\sqrt{2 \times \pi}}$$

Principal = Notional principal of the cap
$\text{PVf}_{\text{Mat.}}$ = Present value factor through the end of the forward 3-month or 6-month period
FRA = The market value of the floating rate for the relevant 3-month or 6-month period
exp(...) = The exponent function, also expressed as *e.*
σ = Annualized 3- or 6-month LIBOR volatility
t = Time to expiry for each cap or floor expressed as a fraction of a year

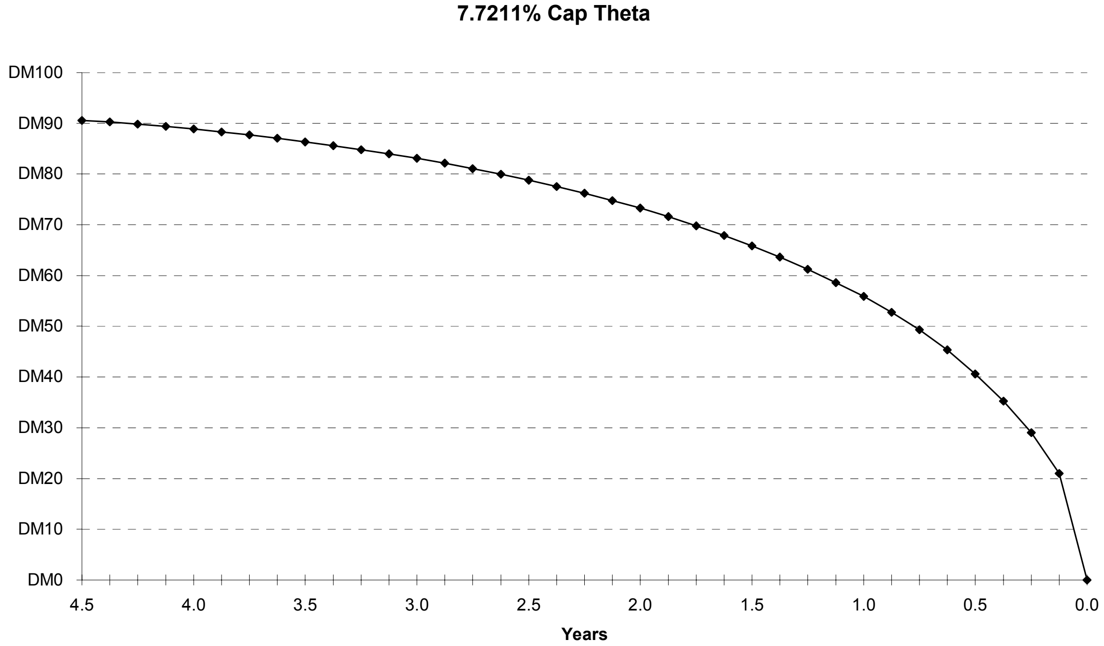
Vega is greatest for at the money options. The profile of vega vs. the FRA is similar to the gamma profile.



Theta

Theta describes changes in the value of the call as time passes. The primary relationship is that the value **decays** with each passing day.

A graph of the cap price relative to the time remaining to expiry shows the relationship very clearly:



Theta is the rate of change with respect to time of the option premium, with all else held constant.

Because an option is a decaying asset, theta will be negative most of the time.

OPTION PORTFOLIO ANALYSIS

Positions

Imagine you have the following positions on your (very simple) option book. All amounts are in thousands of DM:

Instrument	Position	Strike	Maturity	Face Value	Value
Cap #1	Short	7.50%	3	(DM15,000)	(DM141.60)
Cap #2	Long	8.50%	5	DM25,000	DM313.74
Floor #1	Long	6.50%	4	DM20,000	DM226.22
Floor #2	Short	6.25%	3	(DM50,000)	(DM325.05)
Pay Swaption	Short	7.50%	1×2	(DM25,000)	(DM180.57)
Rec. Swaption	Short	8.00%	2×3	(DM50,000)	(DM760.07)
Cash				DM1,000	DM1,000.00
TOTAL					**DM132.67**

An option pricing model tells you that the positions have the following sensitivities:

Instrument	Value	Δ	Γ	Vega	θ
Cap #1	(DM141.60)	(DM1.2197)	(DM0.0036)	(DM9.75)	DM0.14
Cap #2	DM313.74	DM2.5850	DM0.0074	DM34.07	(DM0.27)
Floor #1	DM226.22	(DM2.1518)	DM0.0075	DM19.02	(DM0.26)
Floor #2	(DM325.05)	DM3.6132	(DM0.0153)	(DM28.67)	DM0.55
Pay Swaption	(DM180.57)	(DM1.9387)	(DM0.0059)	(DM11.16)	DM0.26
Rec. Swaption	(DM760.07)	DM5.0098	(DM0.0121)	(DM44.47)	DM0.46
Cash	DM1,000.00	DM0.0000	DM0.0000	DM0.00	DM0.00
TOTAL	**DM132.67**	**DM5.90**	**(DM0.02)**	**(DM40.97)**	**DM0.88**

The delta of the above position is not great. It tells us that the portfolio will lose about 5% of its value for each basis point move down in the yield curve. Clearly for large moves of interest rates this is risky.

Gamma is also working against us. As rates move lower, the loss from the delta will grow faster and faster. It will accelerate at DM0.02 per 0.01% of change in rates.

The vega position exposes us to changing volatility. We are short volatility, so any increase in volatility will cost us money at a rate of DM40.97 per 1% change in volatility.

Fionally, since we have mostly sold options, our liability for them decays at about DM0.88 per day assuming nothing else changes.

Mapping Cash Flows

The process of mapping the portfolio allows us to measure the exposure we have to changes in the underlying market input rates.

Liquid Market Rates

Exchange-traded money market futures (contracts on 3-month LIBOR such as the Eurodollar, Euromark, etc.) are the most liquid market instruments the trader can use to adjust the value sensitivity of the portfolio.

The DM yield curve can thus be defined by using the first 6 contracts of Euromark futures traded on LIFFE followed by the interest rate swaps out to 10 years.

To manage a portfolio, it is vital to know the sensitivity of the portfolio to movements in the liquid instruments which can be used to adjust it. To change the rate sensitivity, the book manager must trade, and he will always prefer to trade the most liquid instruments. Therefore, we want to price and measure sensitivity in terms of the same liquid instruments.

What is required is a table of sensitivities:

Instrument	Interest Rate Portfolio Sensitivity
Mar 95 Future	?
Jun 95 Future	?
Sep 95 Future	?
Dec 95 Future	?
Mar 96 Future	?
Jun 96 Future	?
2-Year Swap Rate	?
3-Year Swap Rate	?
4-Year Swap Rate	?
5-Year Swap Rate	?

If the above table is defined, a position in the most liquid instrument can be used to offset the risk.

As such the trader can pick and choose at what points along the yield curve to remove exposure by trading in those particular instruments.

In addition, new customer business (swaps, options, etc.) can be analyzed by the trader and the effect on the risk of the current book can be easily identified.

A trader has two ways to manage the position and risk of the portfolio:

1. Use exchange traded futures or interbank derivatives (swaps, caps, floors, etc.)
2. Execute customer transactions

Exchange traded futures and interbank trading both have a cost: For futures it is the commission cost and margin calls. For trading in the interbank market, the cost is the bid/offer spread.

If the trader can execute a deal with a customer that moves the risk of the book in a desirable direction the pricing may be slightly better (for the book) than the market quote.

The trader can afford to give better pricing because the alternative is to execute a transaction to hedge certain exposures which will cost money. Some of that savings can be passed onto the customer.

If a customer trade would change the sensitivity of the portfolio to an undesirable level, the trader may quote a price worse than the market.

If the customer trade is executed, the trader will have to go out and pay to execute transactions which will return the book to the desirable position.

Delta
We can delta hedge the portfolio by taking positions to offset the portfolio's base exposure to 0.01% changes in the market input rates. First we have to measure it.

Following is the change in the price of the first cap when each of the input rates is changed by 1 basis point up and down.

Cap #1		**Rates Up 0.01%**	**Rates Down 0.01%**	**Average Change**
Input Rates				
Mar-95	94.83	DM0.18	(DM0.18)	DM0.18
Jun-95	94.49	DM0.16	(DM0.16)	DM0.16
Sep-95	94.10	DM0.15	(DM0.16)	DM0.16
Dec-95	93.70	DM0.11	(DM0.11)	DM0.11
Mar-96	93.36	DM0.05	(DM0.05)	DM0.05
Jun-96	93.08	DM0.03	(DM0.03)	DM0.03
2 Yr	6.4450%	DM0.28	(DM0.32)	DM0.30
3 Yr	6.8450%	(DM2.16)	DM2.13	(DM2.15)
4 Yr	7.0650%	DM0.00	DM0.00	DM0.00
5 Yr	7.2150%	DM0.00	DM0.00	DM0.00

If we measure the sensitivity of each instrument to 0.01% changes in the input rates, we obtain the following table of sensitivities for our portfolio:

Input Rates		**Portfolio Sensitivities**
Mar-95	94.83	DM0.1738
Jun-95	94.49	DM0.3972
Sep-95	94.10	DM0.5664
Dec-95	93.70	DM0.3128
Mar-96	93.36	DM0.0267
Jun-96	93.08	DM0.0147
2 Yr	6.4450%	(DM4.1043)
3 Yr	6.8450%	(DM2.7138)
4 Yr	7.0650%	(DM1.9659)
5 Yr	7.2150%	DM13.5947

These represent the delta of the portfolio to 0.01% changes in the market rates used to construct the pricing curve. Positive numbers mean a gain if the market rate rises.

If the price of the March 1995 futures contract moves down by 1 tick, i.e. if the rate implied by it (5.17%) moves up by 0.01%, the portfolio will gain DM0.1738.

Most of the delta exposure is to the 5-year swap rate. If it moves up by 0.01%, the portfolio will gain DM13.5947.

To "delta hedge" the portfolio, we need to know the sensitivities of each of the underlying input instruments to a one basis point change, too.

The futures are easy. Each basis point is worth the tick value, which is DM25. For the par swaps we measure the sensitivity by revaluing each par swap for a 0.01% change in the par swap rates. The numbers in both cases assume a unit value of DM1 mio.

Input Rates		**Market Equivalents**
Mar-95	94.83	(DM25)
Jun-95	94.49	(DM25)
Sep-95	94.10	(DM25)
Dec-95	93.70	(DM25)
Mar-96	93.36	(DM25)
Jun-96	93.08	(DM25)
2 Yr	6.4450%	(DM182.77)
3 Yr	6.8450%	(DM264.65)
4 Yr	7.0650%	(DM340.59)
5 Yr	7.2150%	(DM410.94)

Both futures and swaps lose value as rates rise. For the futures, this means that the equivalent position is long a futures contract. This position loses value as rates rise.

For the swaps, this is equivalent to receiving fixed. The receiver of fixed is similar to someone long a cash bond: as rates rise the position loses value.

By comparing the basis point sensitivity of the portfolio to that of the input instruments, we can describe how to delta hedge the portfolio:

Input Rates		Portfolio Sensitivities	Market Equivalents	Delta Hedge
Mar-95	94.83	DM0.1738	(DM25)	7.0
Jun-95	94.49	DM0.3972	(DM25)	15.9
Sep-95	94.10	DM0.5664	(DM25)	22.7
Dec-95	93.70	DM0.3128	(DM25)	12.5
Mar-96	93.36	DM0.0267	(DM25)	1.1
Jun-96	93.08	DM0.0147	(DM25)	0.6
2 Yr	6.4450%	(DM4.1043)	(DM182.77)	Pay(DM22.457)
3 Yr	6.8450%	(DM2.7138)	(DM264.65)	Pay(DM10.254)
4 Yr	7.0650%	(DM1.9659)	(DM340.59)	Pay(DM5.772)
5 Yr	7.2150%	DM13.5947	(DM410.94)	RecDM33.082

By buying the indicated number of futures contracts and either receiving or (paying) fixed in the indicated amounts (in DM mio) we can delta hedge the portfolio.

The exposures to the futures contracts will lose money if rates fall (if prices rise). To hedge this we need long positions in the futures.

The calculation of the 5-year par swap hedge is as follows:

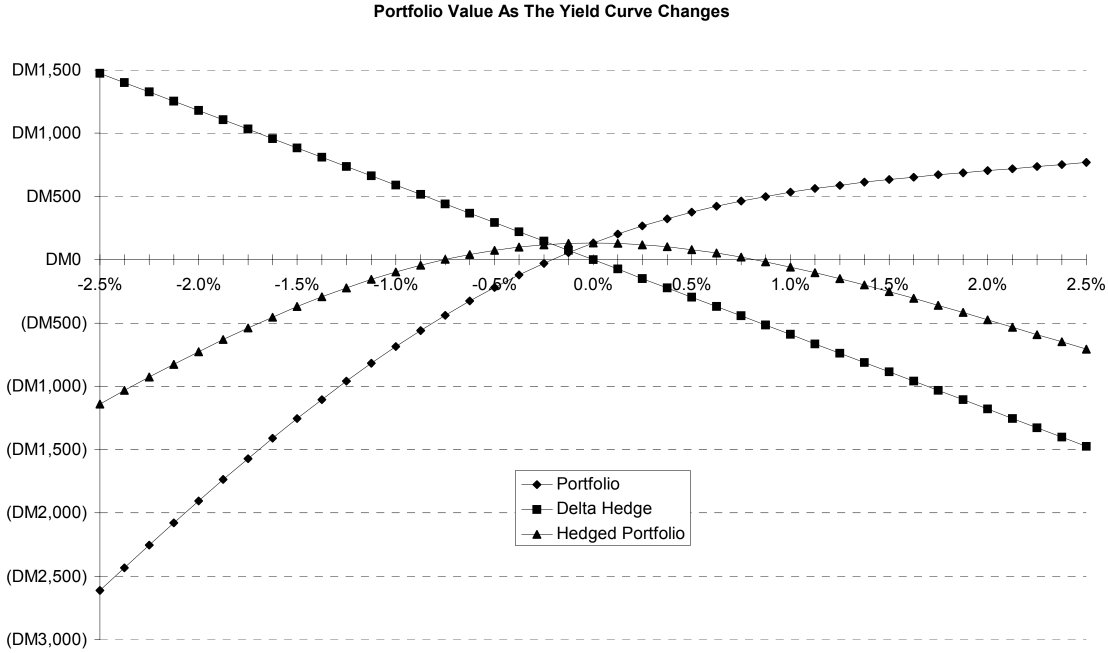
$$\frac{13.5947 \times 1{,}000 \times 1{,}000{,}000}{410.94} = 33{,}082{,}000$$

To hedge the 5-year par swap exposure, we need to receive fixed on a 5-year par swap of face value DM33.082 mio.

The 5-year swap equivalent risk is due to the 2×3 receiver swaption we sold and the 5-year cap #2 we own, which are both tied to changes in the 5-year par swap rate.

Gamma

The delta hedge is only valid for small changes in the yield curve, as can be seen in the following graph.

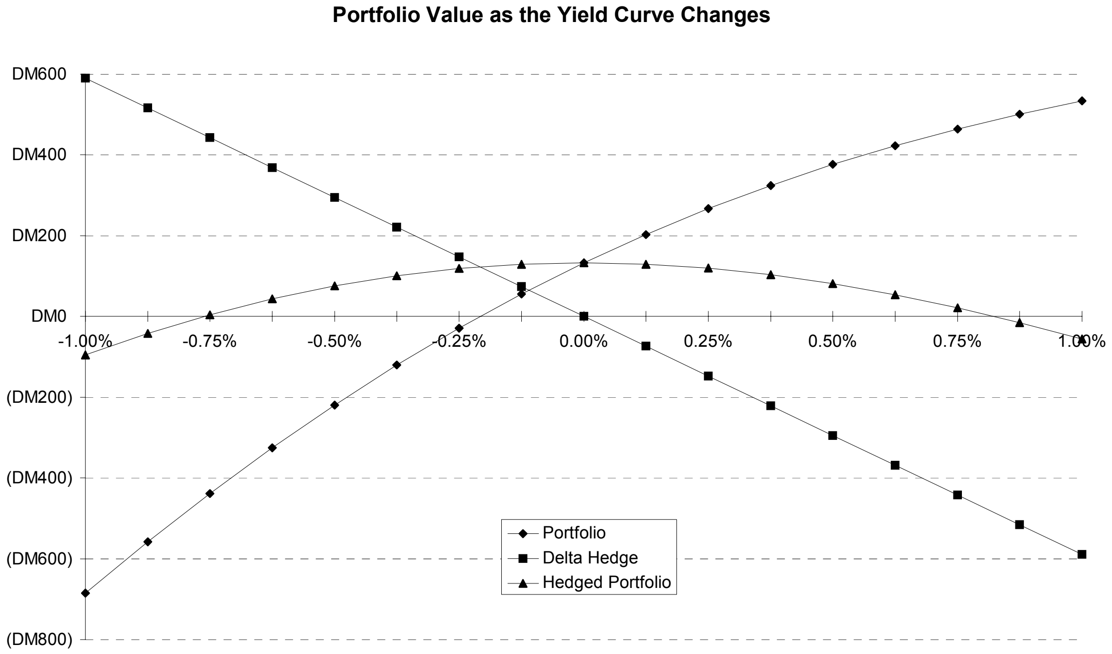


After delta-hedging our portfolio, we are left with a position which will lose money if rates move either up or down.

This is the problem of gamma. The option portfolio has gamma, as can be seen in the graph of its value at a range of possible interest rate change levels. As rates fall, the portfolio loses value at a faster and faster pace.

The futures and swap positions used to create the delta hedge have very little gamma. The delta hedge serves to hedge part of the risk for small movements in interest rates. But the delta hedge falls further and further behind if rates continue to fall.

The effectiveness of the delta hedge for small changes in interest rates can be seen by looking at the middle of the graph above:



For small movements of less than 0.75% in either direction, the delta hedge preserves part of the profit in the option portfolio.

But as rates continue to move, the delta hedge has to be rebalanced or the losses will grow.

The gamma is greatest on the 1×2 payer swaption we sold, and it makes a good example of the gamma risk. To delta hedge it, we need to pay fixed on par swaps.

As rates move down, the owner is less and less likely to exercise, and we need to unwind the hedge, in effect receiving fixed at lower and lower rates.

As rates move up, the owner is more and more likely to exercise, and we need to have more and more hedge, paying fixed at higher and higher rates.

Whichever way rates move, they are moving against us.

Vega

We can map the vega of the portfolio to the underlying cap and swaption volatilities used to price the caps, floors and swaptions. In effect we are creating a "vega map" in terms of the instruments which might be used to hedge it.

Instrument			Cap #1	Cap #2	Floor #1	Floor #2	Pay Swaption	Rec Swaption	TOTAL
Face Value			(DM15,000)	DM25,000	DM20,000	(DM50,000)	(DM25,000)	(DM50,000)	
Value			(DM141.60)	DM313.74	DM226.22	(DM325.05)	(DM180.57)	(DM760.07)	(DM867.33)
Cap Volatility									
	1 Yr	19.50%	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00
	2 Yr	17.80%	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00
	3 Yr	17.20%	(DM9.80)	DM0.00	DM0.00	(DM28.97)	DM0.00	DM0.00	(DM38.76)
	4 Yr	16.20%	DM0.00	DM0.00	DM19.182	DM0.00	DM0.00	DM0.00	DM19.18
	5 Yr	15.50%	DM0.00	DM34.39	DM0.00	DM0.00	DM0.00	DM0.00	DM34.39
Swaption Volatility									
	1×2	17.03%					(DM11.16)	DM0.00	(DM11.16)
	1×4	15.64%					DM0.00	DM0.00	DM0.00
	2×3	14.98%					DM0.00	(DM44.47)	(DM44.47)

In the map above, the vegas are mapped to the term volatilities of the various caps and floors, and swaptions. This is done by varying each volatility input by 1% and recalculating the option prices. Done in this manner, each instrument is sensitive to only one market volatility input.

If we assume that the term structure of volatility shifts up and down in parallel fashion, we can try and hedge the entire vega risk with a single purchased option. We can also hedge the vega of the portfolio to each cap and swaption volatility by using ATM caps, floors and swaption to hedge each bucket separately.

There are other ways to map vega, too. One common approach is to map to the implied forward forward volatilities, which would map part of each cap and floor to various forward forward volatilities.

It is also feasible to tie the swaptions to the map of forward forward volatilities, or even to map the whole portfolio to ATM swaption volatility, to use purchased swaptions to hedge short cap and floor vega.

Theta

The theta of the portfolio is DM0.88. This means that it will gainDM0.88 per day, assuming all else is constant. This will of course gather speed as the various instruments move nearer to expiry.

ADDING A SWAPTION

Now let us imagine we have a client who wishes to purchase DM100 mio of 1×4 payer swaption struck at 7.75%. The 1×4 forward swap rate is currently 7.6482% and the appropriate volatility is 15.64%.

According to the option pricing model, the value of this swaption is DM1,292.19. We agree to charge the client this price and he buys the swaption. What does this do to the portfolio we already had in place?

New Position and Delta Hedge

The portfolio now consists of the following:

Instrument	Position	Strike	Maturity	Face Value	Value
Cap #1	Short	7.50%	3	(DM15,000)	(DM141.60)
Cap #2	Long	8.50%	5	DM25,000	DM313.74
Floor #1	Long	6.50%	4	DM20,000	DM226.22
Floor #2	Short	6.25%	3	(DM50,000)	(DM325.05)
Pay Swaption	Short	7.50%	1×2	(DM25,000)	(DM180.57)
Rec. Swaption	Short	8.00%	2×3	(DM50,000)	(DM760.07)
Pay Swaption	Short	7.75%	1×4	(DM100,000)	(DM1,292.19)
Cash				DM2,523	DM2,292.19
TOTAL					**DM132.67**

An option pricing model tells you that the positions have the following sensitivities:

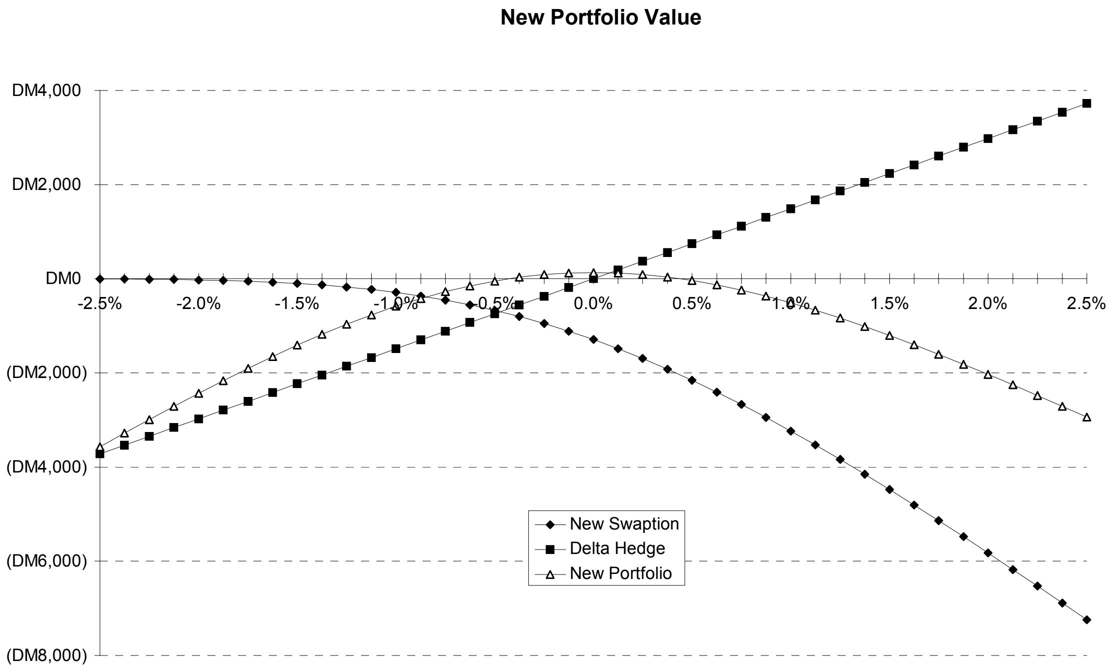
Instrument	Value	Δ	Γ	Vega	θ
Cap #1	(DM141.60)	(DM1.2197)	(DM0.0036)	(DM9.75)	DM0.14
Cap #2	DM313.74	DM2.5850	DM0.0074	DM34.07	(DM0.27)
Floor #1	DM226.22	(DM2.1518)	DM0.0075	DM19.02	(DM0.26)
Floor #2	(DM325.05)	DM3.6132	(DM0.0153)	(DM28.67)	DM0.55
Pay Swaption	(DM180.57)	(DM1.9387)	(DM0.0059)	(DM11.16)	DM0.26
Rec. Swaption	(DM760.07)	DM5.0098	(DM0.0121)	(DM44.47)	DM0.46
Pay Swaption	(DM1,292.19)	(DM14.8823)	(DM0.0498)	(DM91.49)	DM1.95
Cash	DM1,000.00	DM0.0000	DM0.0000	DM0.00	DM0.00
TOTAL	**DM132.67**	**(DM8.9845)**	**(DM0.0717)**	**(DM132.46)**	**DM2.83**

The delta has shifted from positive to negative, which means the overall position is now equivalent to paying fixed on a lot of swaps. The gamma has jumped, and will be working even more against us if rates move much at all.

The delta map now looks like this:

Input Rates		New Swaption	Delta Hedge	New Portfolio	New Hedge
Mar-95	94.83	DM1.34	53.4	DM1.5100	60.4
Jun-95	94.49	DM1.24	49.6	DM1.6382	65.5
Sep-95	94.10	DM1.24	49.6	DM1.8061	72.2
Dec-95	93.70	DM0.66	26.3	DM0.9701	38.8
Mar-96	93.36	DM0.00	0.0	DM0.0267	1.1
Jun-96	93.08	DM0.00	0.0	DM0.0147	0.6
2 Yr	6.4450%	DM0.03	DM0.158	(DM4.0755)	(DM22.299)
3 Yr	6.8450%	DM0.04	DM0.167	(DM2.6697)	(DM10.087)
4 Yr	7.0650%	DM0.06	DM0.178	(DM1.9053)	(DM5.594)
5 Yr	7.2150%	(DM19.28)	(DM46.920)	(DM5.6865)	(DM13.838)

Assuming we adjust our delta hedge to the amounts in the far right column, our portfolio now faces interest rate exposure even greater than before:



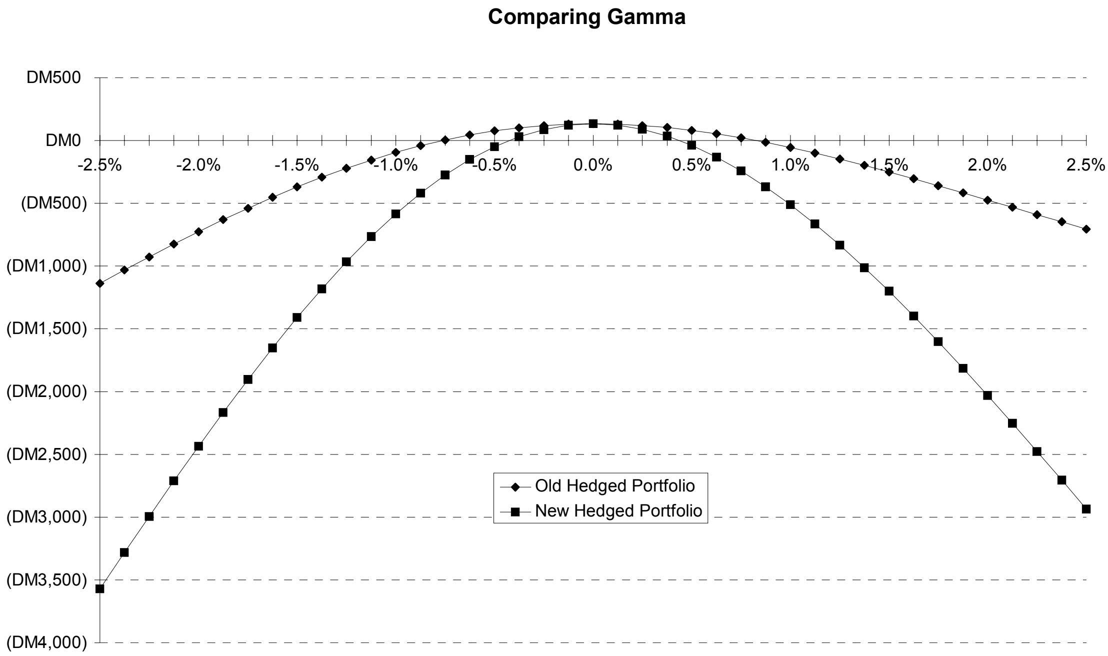
The new swaption will cost us money if rates rise, as its owner will be more and more likely to exercise into more and more profit (for him).

The delta hedge of the new swaption is paying fixed on DM47 mio of 5-year swap offset by buying large amounts of the four nearby futures. The combined effect is to leave us with a large amount of gamma.

Gamma Effect

The change in the gamma of the portfolio is from DM0.02 per basis point of rate movement to DM0.07 per basis point of rate movement.

Since we sold more options, gamma works against us harder than ever. The change is easy to see when we compare the delta-hedged portfolio value across a range of interest rate changes before and after the new swaption:



For option books consisting mostly of sold options, gamma is highly problematic.

There is only one way to hedge it: buy options sensitive to the same interest rate changes.

Hedging Vega

In order to hedge this option book more effectively, we need to buy options. Another approach used by many market participants is to begin with the vega, offsetting that and then hedging the residual delta risk.

This approach has one major positive: it also usually offsets the gamma risk.

It has one big negative, too: it means buying large quantities of options from the market. If client business is strong on both sides, i.e. clients are both buying and selling options to the bank, bid-offer spreads can mean nice profits with far less risk. If clients are mostly buying options from the bank, and the bank has to buy its hedges from the market, the profit of the client business will be far smaller.

In this case, let us imagine we wish to buy the same 1×4 payer swaptions to hedge the vega. How much do we need to buy?

The vega of the 1×4 payer swaption was DM91.49 per DM100,000. As a percent of face value, vega is thus: $\frac{91.49}{100{,}000} = 0.09149\%$.

The vega in the new portfolio is (DM132.46). To offset this much vega, we need to buy DM144,783 worth of the 1×4 payer swaption:

$$\text{Hedge} = \frac{132.46}{0.09149\%} = 144{,}783$$

At a price of 1.29219%, this will cost DM1,871. The portfolio now consists of:

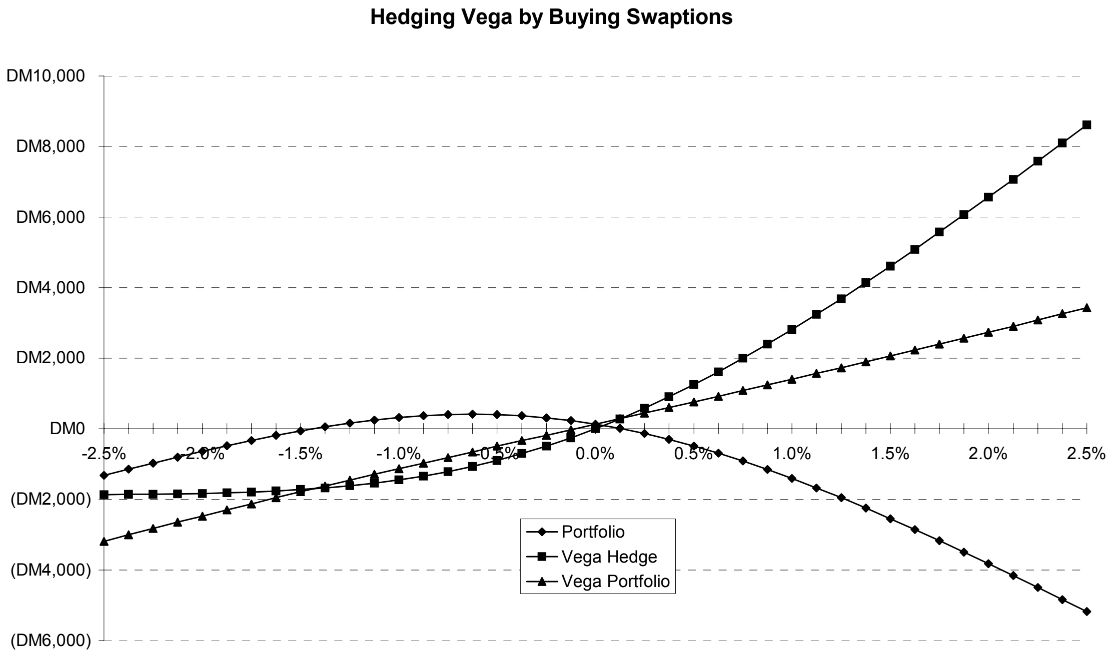
Instrument	Position	Strike	Maturity	Face Value	Value
Cap #1	Short	7.50%	3	(DM15,000)	(DM141.60)
Cap #2	Long	8.50%	5	DM25,000	DM313.74
Floor #1	Long	6.50%	4	DM20,000	DM226.22
Floor #2	Short	6.25%	3	(DM50,000)	(DM325.05)
Pay Swaption	Short	7.50%	1×2	(DM25,000)	(DM180.57)
Rec. Swaption	Short	8.00%	2×3	(DM50,000)	(DM760.07)
Pay Swaption	Short	7.75%	1×4	DM44,783	DM578.68
Cash				DM421	DM421.32
TOTAL					**DM132.67**

An option pricing model tells you that the positions have the following sensitivities:

Instrument	Value	Δ	Γ	Vega	θ
Cap #1	(DM141.60)	(DM1.2197)	(DM0.0036)	(DM9.75)	DM0.14
Cap #2	DM313.74	DM2.5850	DM0.0074	DM34.07	(DM0.27)
Floor #1	DM226.22	(DM2.1518)	DM0.0075	DM19.02	(DM0.26)
Floor #2	(DM325.05)	DM3.6132	(DM0.0153)	(DM28.67)	DM0.55
Pay Swaption	(DM180.57)	(DM1.9387)	(DM0.0059)	(DM11.16)	DM0.26
Rec. Swaption	(DM760.07)	DM5.0098	(DM0.0121)	(DM44.47)	DM0.46
Pay Swaption	DM578.68	DM6.6647	DM0.0223	DM40.97	(DM0.87)
Cash	DM421.32	DM0.0000	DM0.0000	DM0.00	DM0.00
TOTAL	**DM132.67**	**DM12.5626**	**DM0.0004**	**DM0.00**	**DM0.01**

This is a very interesting result. We have a lot of delta risk, but vega is down to zero, and the gamma is nearly all offset, as is theta.

This is shown on the graph following:

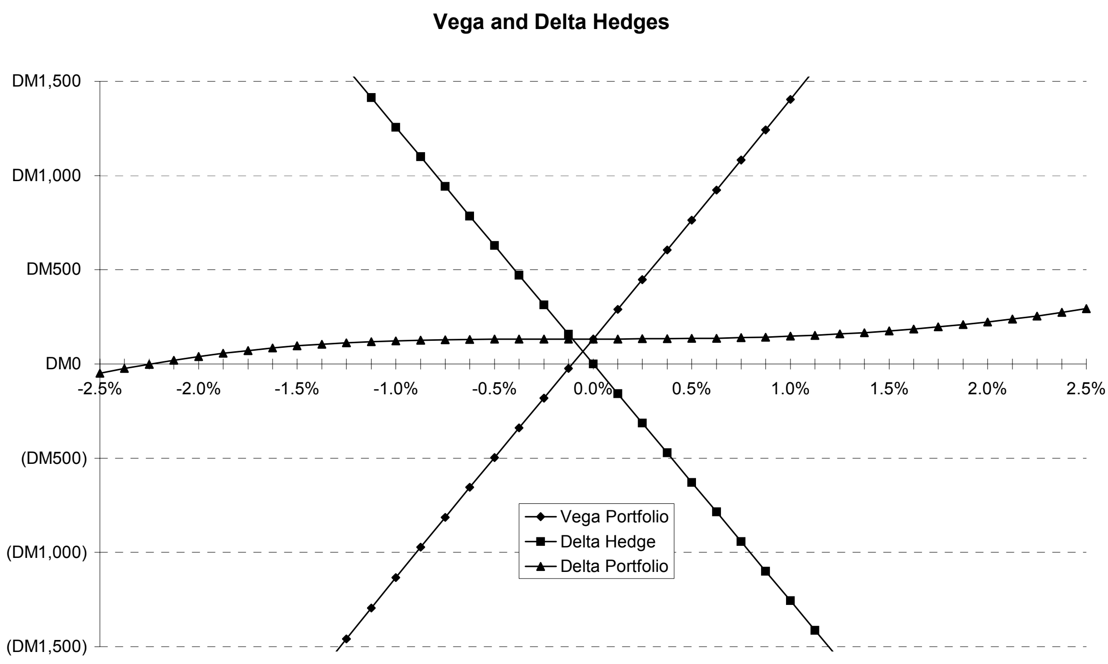


All that remains is to hedge the delta and see where we stand for various interest rate movements.

Hedging Delta
To hedge the delta, we need to map the delta to the market input instruments once more.

Input Rates		Previous Portfolio	New Vega Swaption	New Portfolio	New Delta Hedge
Mar-95	94.83	DM0.1738	(DM1.9346)	(DM1.7608)	-70.4
Jun-95	94.49	DM0.3972	(DM1.7967)	(DM1.3994)	-56.0
Sep-95	94.10	DM0.5664	(DM1.7949)	(DM1.2285)	-49.1
Dec-95	93.70	DM0.3128	(DM0.9516)	(DM0.6388)	-25.6
Mar-96	93.36	DM0.0267	DM0.0000	DM0.0267	1.1
Jun-96	93.08	DM0.0147	DM0.0000	DM0.0147	0.6
2 Yr	6.4450%	(DM4.1043)	(DM0.0418)	(DM4.1461)	(DM22.685)
3 Yr	6.8450%	(DM2.7138)	(DM0.0638)	(DM2.7776)	(DM10.495)
4 Yr	7.0650%	(DM1.9659)	(DM0.0878)	(DM2.0537)	(DM6.030)
5 Yr	7.2150%	DM13.5947	DM27.9159	DM41.5105	DM101.014

Assuming that is done and we adjust the delta hedge once more, we can measure the overall position across a range of interest rates:



Gamma

If we are willing to invest in the large payer swaption, gamma is hedged while we are hedging volatility.

Hedging delta only is better than not hedging at all. The same is true for hedging vega.

Hedging delta alone often increases gamma and vega.

Hedging vega and delta together may be a far more efficient way to protect the book's profits.

A well-constructed hedge of vega and gamma together is normally far more stable than hedging either one separately as both volatility and interest rate move.

No hedge is permanent. New deals are being booked all the time. Liquidity of futures, swaps, OTC options and futures options (where available), combined with a diversity of client deals leads to greater profitability from the option business.